Stripe Picture Languages

Definition: A picture languages L is called a stripe picture language, if there are real numbers k, d_1 and d_2 such that, for any picture $p \in L$ and any point $(m,n) \in V(p)$,

$$km + d_1 \leq n \leq km + d_2.$$

Lemma: Let $G = (N, \pi, P, S)$ be a reduced context-free grammar such that $dccp(G)$ is a (k, d_1, d_2)-stripe picture. For any non-empty word x such that $A \Longrightarrow^* xAy$ or $A \Longrightarrow^* yAx$ for some $A \in N$ and $y \in \pi^*$, sh(x)=(0,0)$or sh(x) = (m, km)$ for some m.

Corollary: Any (k, d_1, d_2)-stripe picture language with a non-rational k is finite.

String Representation Lemma

Lemma: Let k be a rational number and d_1 and d_2 real numbers with $d_1 < d_2$. Then there is an alphabet V and an encoding μ which maps any picture in the stripe to a word over V such that the following properties hold:

i) For two (k, d_1, d_2)-stripe pictures q and q', $\mu(q) = \mu(q')$ if and only if $q = q'$.

ii) For a (k, d_1, d_2)-stripe picture q, $\mu(q)$ can be computed in linear time (in the size of q).

iii) If L is a regular (k, d_1, d_2)-stripe picture language, then $\mu(L) = \{\mu(q) \mid q \in L\}$ is a regular word language over V, which can effectively be constructed.

Decision Problems for Stripe Picture Grammars

Theorem: For a regular grammar G such that $bccp(G)$ is a stripe picture language and a picture p, it is decidable in linear time whether or not $p \in bccp(G)$.

Theorem: For two regular grammars G_1 and G_2 such that $bccp(G_1)$ and $bccp(G_2)$ are stripe picture languages, it is decidable whether or not $bccp(G_1) = bccp(G_2)$.

Theorem: For a regular grammar G such that $bccp(G)$ is a stripe picture language, it is decidable whether or not $bccp(G)$ contains
i) a simple curve,
ii) a closed simple curve,
iii) a regular curve,
iv) an Eulerian curve.

Decidability of Being a Stripe Language

Lemma: Let $G = (N, \pi, P, S)$ be a context-free grammar in Chomsky normal form such that $L(G) = pref(L(G))$. If there is a real number k such that, for all words x in the set

$$T = \{x \mid x \in \pi^*,\ A \Longrightarrow^* xAy \text{ or } A \Longrightarrow^* yAx \text{ for some } y \in \pi^* \text{ and } A \in N\},$$

$sh(x) = (m, mk)$ for some m, then there are two real numbers d_1 and d_2 such that $dccp(G)$ is a (k, d_1, d_2)-stripe language.

Theorem: For a context-free grammar G, it is decidable whether or not $bccp(G)$ is a stripe language.

Extended Chain Code Pictures

Inductive definition of an extended drawn picture $edccp(w)$ of $w \in \pi_{\updownarrow}$

— if $w = \lambda$, then $edccp(w) = ((0,0), \emptyset, (0,0), \downarrow)$,
— if $w = w'b$, $w' \in \pi_{\updownarrow}^*$, $b \in \pi$ and $edccp(w') = ((0,0), p, z, \downarrow)$,
then $edccp(w) = ((0,0), p \cup \{z, b(z))\}, b(z), \downarrow)$,
— if $w = w'b$, $w' \in \pi_{\updownarrow}^*$, $b \in \pi$ and $edccp(w') = ((0,0), p, z, \uparrow)$,
then $edccp(w) = ((0,0), p, b(z), \uparrow)$, — if $w = w'b$, $w' \in \pi_{\updownarrow}^*$, $b = \uparrow$, $edccp(w') = ((0,0), p, z, s)$ and $s \in \{\uparrow, \downarrow\}$,
then $edccp(w) = ((0,0), p, z, \uparrow)$, — if $w = w'b$, $w' \in \pi_{\updownarrow}^*$, $b = \downarrow$, $edccp(w') = ((0,0), p, z, s)$ and $s \in \{\uparrow, \downarrow\}$,
then $edccp(w) = ((0,0), p, z, \downarrow)$.

Theorem: i) $\mathcal{CCP}_{\updownarrow}(REG) \subset \mathcal{CCP}_{\updownarrow}(CF) \subset \mathcal{CCP}_{\updownarrow}(CS) = \mathcal{CCP}_{\updownarrow}(RE)$.
ii) $\mathcal{CCP}(X) \subset \mathcal{CCP}_{\updownarrow}(X)$ for $X \in \{REG, CF, CS, RE\}$.

A First Koch Curve and a Result

$$(\pi, \pi, \{u \to urul^2url, d \to dldr^2dld, r \to rdru^2rdr, l \to luld^2lul\}, urdl)$$

Theorem: $\mathcal{CCP}(CF) \subset \mathcal{CCP}(E0L)$

Turtle Grammars – I

Definition:
A turtle grammar is an $(n+4)$-tuple $G = (V, T, P_1, P_2, \ldots, P_n, w, \alpha_0, \alpha)$, where

— V is a finite alphabet, and $T \subseteq V$ is a subset containing the letter F,
— for $1 \leq i \leq n$, P_i is a finite set of productions of the form $A \to v$ with $A \in V$ and $v \in (V \cup \{+, -\})^*$,
— $w \in (V \cup \{+, -\})^*$,
— α_0 and α are two angles.

Turtle Grammars – II

Let α_0 and α be two angles and, V an alphabet containing the letter F. For a word $w \in (V \cup \{+,-\})^*$, we define inductively a configuration $c(w) = (M, (x,y), \beta)$ with a set M of lines of unit length, a point (x,y) in the plain and an angle as follows

— $c(\lambda) = (\emptyset, (0,0), \alpha_0)$,

— if $c(w) = (M, (x,y), \beta)$, then

- $c(wx) = c(w)$ for $x \in V$ and $x \neq F$,
- $c(w+) = (M, (x,y), \beta + \alpha)$ and
- $c(w-) = (M, (x,y), \beta - \alpha)$,
- $c(wF) = (M \cup \{b\}, (x',y'), \beta)$, where (x',y') is the point such that the distance between (x',y') and (x,y) is 1, b is the line connecting (x,y) and (x',y') and the angle between b and the x-axes is β.

The picture $tur(w)$ is defined as the first component of $c(w)$.

A Second Koch curve

$K_2 = ((\{F\}, \{F \to F + F - -F + F\}, F),\ 0^o,\ 60^o)$
$K_2' = (\{F\}, \{F \to F + [F] - [F]F\}, F,\ 0^0,\ 60^0)$

Turtle Grammars – III

An extended turtle grammar is an $(n+4)$-tuple

$$G = (V, T, P_1, P_2, \ldots, P_n, w, \alpha_0, \alpha),$$

where

— V is a finite alphabet, and $T \subseteq V$ is a subset containing the letter F,

— for $1 \leq i \leq n$, P_i is a finite set of productions of the form $A \to v$ with $A \in V$, $v \in (V \cup \{+, -, [,]\})^*$, and v is correctly bracketed with respect to [and],

— $w \in (V \cup \{+, -, [,]\})^*$ and w is correctly bracketed with respect to [and],

— α_0 and α are two angles.

Turtle Grammars – IV

Let α_0 and α be two angles and, V an alphabet containing F. For a word $w \in (V \cup \{+,-,[,]\})^*$, we define inductively the extended configuration $ec(w) = (M,(x,y),s,\beta_0,\beta)$ with a set M of lines of unit length, a point (x,y) in the plain, a sequence s of angles, and two angles β_0 and β by

— $ec(\lambda) = (\emptyset,(0,0),\lambda,\alpha_0,\alpha_0)$,

— if $ec(w) = (M,(x,y),s,\beta_0,\beta)$, then

- $ec(wx) = c(w)$ for $x \in V$ and $x \neq F$,
- $ec(w+) = (M,(x,y),s,\beta_0,\alpha)$ and $ec(w-) = (M,(x,y),s,\beta_0,-\alpha)$,
- $ec(wF) = (M \cup \{b\},(x',y'),s,\beta_0,\beta)$, where (x',y') is the point such that the distance between (x',y') and (x,y) is 1, b is the line connecting (x,y) and (x',y') and the angle between b and the line given by the angle β_0 is α,
- $ec(w[) = (M,(x,y),s\beta_0,\alpha,0^0)$, and
- $ec(w]) = (M,(x,y),s',\beta,0^0)$ where $s = s'\beta$

The extended picture $etur(w)$ is defined as the first component of $ec(w)$.

A Statement

Theorem:

For a picture language L, the following two statements are equivalent:

i) $L = bccp(G)$ for some ET0L system G.

ii) $L = etur(G)$ for some (G, α_0, α) where G is an ET0L system, $\alpha = 90^o$ and α is a multiple of 90^o.