Multisets

multiset M over V — function from V^* into $\mathbf{N}$

$M(x)$ — multiplicity of $x \in V^*$

A multiset M is called finite if there is a finite subset U of V^* such that $M(x) = 0$ for all $x \notin U$.

Cardinality — $\#(M) = \sum_{x \in V^*} M(x)$,
Length — $l(M) = \sum_{x \in V^*} M(x)|x|$

a finite multiset can be represented as $[w_1, w_2, \ldots, w_n]$,
$l(M) = |w_1 w_2 \ldots w_n|$,
$\#_a(M) = \#_a(w_1 w_2 \ldots w_n)$

Multiset Splicing I

Definition: For multisets $M = [w_1, w_2, \ldots, w_n]$ and $M' = [v_1, v_2, \ldots, v_n]$ of words over V and a set P of splicing rules over V, we define

— a sequential derivation step $M \underset{s}{\Longrightarrow} M'$ by $[w_1, w_2] \underset{p}{\Longrightarrow} [v_1, v_2]$ and $w_i = v_i$ for $3 \leq j \leq n$ for some $p \in P$ and some appropriate order of the elements in M and M',

— a maximally parallel derivation step $M \underset{mp}{\Longrightarrow} M'$ by $[w_{2i-1}, w_{2i}] \underset{p_i}{\Longrightarrow} [v_{2i-1}, v_{2i}]$ for $1 \leq i \leq k \leq \frac{n}{2}$ and $w_i = v_i$ for $2k + 1 \leq j \leq n$ for some $p_i \in P$ and some appropriate order of the elements in M and M', and by the requirement that there is no multiset $[w, w'] \subseteq [w_{2k+1}, w_{2k+2}, \ldots, w_n]$ to which a splicing rule $p \in P$ can be (successfully) applied,

– a strongly maximally parallel derivation step $M \underset{smp}{\Longrightarrow} M'$ by $M \underset{mp}{\Longrightarrow} M'$ for some k (as in the preceding item) and there is no M'' with $M \underset{mp}{\Longrightarrow} M''$ for some $k' > k$.

Multiset Splicing Systems

Definition: A multiset splicing system is a triple $G = (V, P, M)$ where
– V is an alphabet,
– P is a finite set of splicing rules over V such that, for any rule $r_1 \# r_2 \$ r_3 \# r_4 \in P$, $r_i \neq \lambda$ for $1 \leq i \leq 4$, and
– M is a finite multiset over V.

Definition We define the sequential, maximally parallel and strongly maximally parallel multiset languages $mL(G, s)$, $mL(G, mp)$ and $mL(G, smp)$ generated by G as

$$\begin{aligned} mL(G, s) &= \{K \mid M \Longrightarrow_s^* K\}, \\ mL(G, mp) &= \{K \mid M \Longrightarrow_{mp}^* K\}, \\ mL(G, smp) &= \{K \mid M \Longrightarrow_{smp}^* K\}. \end{aligned}$$

Some Notations

for $Y \in \{s, mp, smp\}$, we denote by

$m\mathcal{L}(Y)$	–	the family of all languages $mL(G, Y)$ which can be generated by a multiset splicing system G in the derivation mode Y,
$m\mathcal{L}_n(Y)$	–	the family of all languages $mL(G, Y)$ which can be generated by a multiset splicing system $G = (V, P, M)$ with $\#(M) = n$ in the derivation mode Y.

Some Facts I

Lemma:
For any multiset splicing system $G = (V, P, M)$, $a \in V$, $Y \in \{s, mp, smp\}$, and any $K \in mL(G, Y)$,

$$\#(K) = \#(M), \ l(K) = l(M), \text{ and } \#_a(K) = \#_a(M).$$

Theorem:
For two integers n and m, $m \neq n$, and two derivation modes $Y \in \{s, mp, smp\}$ and $Y' \in \{s, mp, smp\}$, the language families $m\mathcal{L}_n(Y)$ and $m\mathcal{L}_m(Y')$ are incomparable.

Some Facts II

Theorem:

i) For $n \in \{1, 2, 3\}$, $m\mathcal{L}_n(s) = m\mathcal{L}_n(mp) = m\mathcal{L}_n(smp)$.

ii) For $n \geq 4$, $m\mathcal{L}_n(mp)$ and $m\mathcal{L}_n(smp)$are both incomparable to $m\mathcal{L}_n(s)$.

iii) For $n \geq 5$, $m\mathcal{L}_n(mp)$ is not contained in $m\mathcal{L}_n(smp)$.

iv) For $n \geq 6$, the classes $m\mathcal{L}_n(s)$, $m\mathcal{L}_n(mp)$, and $m\mathcal{L}_n(smp)$ are pairwise incomparable.

Sticking Operation – Prolongation to the Rigth

For $x \in W_\varrho(V)$ mit $x = x_1x_2x_3$, $y \in W_\varrho(V)$, we define $\mu_r(x,y)$ by

1. $x_1x_2 \begin{bmatrix} u \\ v \end{bmatrix} y'$, if $x_3 = \binom{u}{\lambda}$, $y = \binom{\lambda}{v} y'$ $(u, v \in V^*$, $y' \in R_\varrho(V)$,
2. $x_1x_2 \begin{bmatrix} u \\ v \end{bmatrix} y'$, if $x_3 = \binom{\lambda}{v}$, $y = \binom{u}{\lambda} y'$ $(u, v \in V^*$, $y' \in R_\varrho(V))$,
3. $x_1x_2 \begin{bmatrix} u \\ v \end{bmatrix} \binom{u'}{\lambda}$, if $x_3 = \binom{uu'}{\lambda}$, $y = \binom{\lambda}{v}$ $(u, v, u' \in V^*, y' \in R_\varrho(V))$,
4. $x_1x_2 \begin{bmatrix} u \\ v \end{bmatrix} \binom{\lambda}{v'}$, if $x_3 = \binom{u}{\lambda}$, $y = \binom{\lambda}{vv'}$ $(u, v, v' \in V^*$, $y' \in R_\varrho(V))$,
5. $x_1x_2 \binom{uv}{\lambda}$, if $x_3 = \binom{u}{\lambda}$, $y = \binom{v}{\lambda}$ $(u, v \in V^*)$,
6. $x_1x_2 \begin{bmatrix} v \\ u \end{bmatrix} \binom{\lambda}{u'}$, if $x_3 = \binom{\lambda}{uu'}$, $y = \binom{v}{\lambda}$ $(u, v, u' \in V^*)$,
7. $x_1x_2 \begin{bmatrix} v \\ u \end{bmatrix} \binom{v'}{\lambda}$, if $x_3 = \binom{\lambda}{u}$, $y = \binom{vv'}{\lambda}$ $(u, v, v' \in V^*)$,
8. $x_1x_2 \binom{uv}{\lambda}$, if $x_3 = \binom{\lambda}{u}$, $y = \binom{\lambda}{v}$ $(u, v \in V^*)$.

Sticker Systems

Definition: i) A *sticker system* is a quadruple $G = (V, \varrho, A, D)$ where

– V is an alphabet,

– $\varrho \subset V \times V$ is a symmetric relation on V,

– A is a finite subset of $LR_\varrho(V)$, and

– D is a finite subset of $W_\varrho(V) \times W_\varrho(V)$.

ii) We say that $y \in LR_\varrho(V)$ is derived by $x \in LR_\varrho(V)$ in one step (written as $x \Longrightarrow y$) iff $y = \mu_l(\mu_r(x, y_2), y_1)$ for some $(y_1, y_2) \in D$.

By $\stackrel{*}{\Longrightarrow}$ we denote the reflexive and transitive closure of $\Longrightarrow$.

iii) The molecule language $ML(G)$ and the word language $wL(G)$ generated by G are defined by

$$ML(G) = \{z \mid x \Longrightarrow s{*}z,\ x \in A,\ z \in \begin{bmatrix} V \\ V \end{bmatrix}_\varrho^+\}$$

and

$$wL(G) = \{w \mid \begin{bmatrix} w \\ v \end{bmatrix} \in ML(G) \text{ for some } v \in V^+\},$$

respectively.

Special Sticker Systems I

Definition:

A sticker system $G = (V, \varrho, A, D)$ is called

- *one-sided* if, for each pair $(u, v) \in D$, $u = \binom{\lambda}{\lambda}$ or $v = \binom{\lambda}{\lambda}$ hold,
- *regular* if, for each pair $(u, v) \in D$, $u = \binom{\lambda}{\lambda}$ holds,
- *simple* if, for each pair $(u, v) \in D$, $uv \in \binom{V^*}{\lambda}$ or $uv \in \binom{\lambda}{V^*}$ hold.

Special Sticker Systems II

Definition:
i) For a sticker system $G = (V, \varrho, A, D)$ and a natural number $d \geq 1$, we define the language $ML_d(G)$ as the set of all molecules which have a derivation

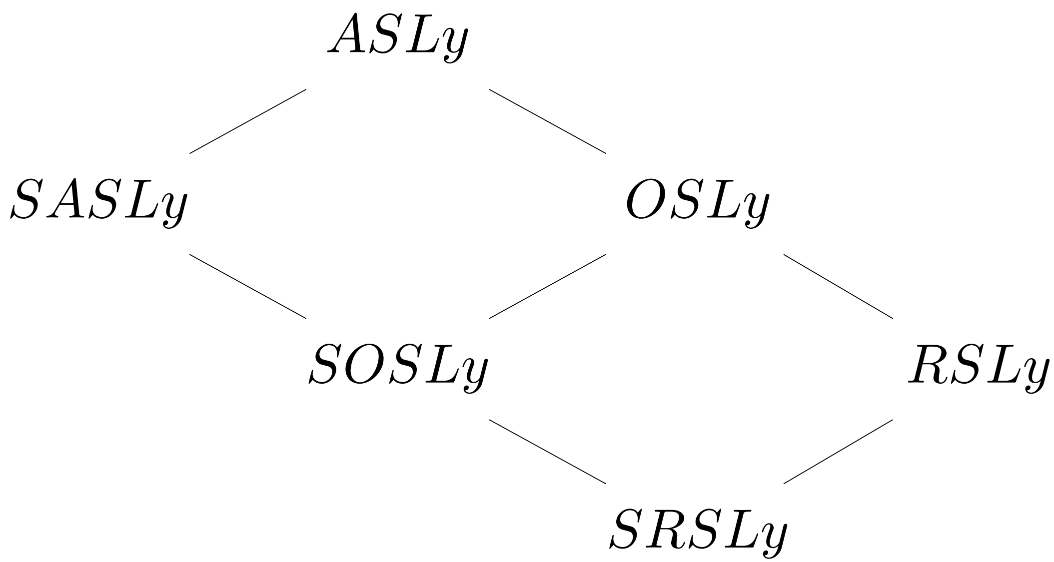
$$x = x_0 \Longrightarrow x_1 \Longrightarrow \ldots \Longrightarrow x_k \text{ with } x_k \in \begin{bmatrix} V \\ V \end{bmatrix}_\varrho^* \text{ and } d(x_i) \leq d \text{ for } 0 \leq i \leq k.$$

ii) We say that a molecule language $L \subset \begin{bmatrix} V \\ V \end{bmatrix}_\varrho^*$ or a word language $L' \subset V^*$ can be generated with *bounded delay*, if there are a sticker system $G = (V, \varrho, A, D)$ and a natural number $d \geq 1$ such that $ML(G) = ML_d(G)$ and $L = ML(G)$ and $L' = wL(G)$, respectively, are valid.

Sticker System – Results I

Lemma: For $X \in \{A, O, R, SA, SO, SR\}$, $XSL(b) \subseteq XSL$.

Lemma: For $y \in \{(b), \lambda\}$, the following diagram holds.



Sticker Systems – Results II

Lemma: $ASL \subseteq \mathcal{L}(CS)$.

Lemma: $OSL \subseteq \mathcal{L}(REG)$.

Lemma: $SOSL(b) = SOSL$ und $SRSL(b) = SRSL$.

Lemma: $\mathcal{L}(REG) \subseteq RSL(b)$.

Lemma: $ASL(b) = \mathcal{L}(LIN)$.

Lemma: There is a regular language which is not in $SOSL$.

Sticker Systems – Results III

Satz: The following diagram holds.