

# Logik für Informatiker

## Logic for computer scientists

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# The Boolean Connectives

## Negation — Truth table

$P$	$\neg P$
TRUE	FALSE
FALSE	TRUE

# The Henkin-Hintikka game

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"Checkmate!"

# The Henkin-Hintikka game

Is a sentence true in a given world?

- Players: **you** and the **computer** (Tarski's world)
- You claim that a sentence is true (or false), Tarski's world will claim the opposite
- In each round, the sentence is **reduced** to a simpler one
- When an **atomic sentence** is reached, its truth can be directly inspected in the given world

You have a **winning strategy** exactly in those cases where your claim is **correct**.

## Negation — Game rule

Form	Your commitment	Player to move	Goal
$\neg P$	either	—	Replace $\neg P$ by $P$ and switch commitment

## Conjunction — Truth table

P	Q	$P \wedge Q$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

## Conjunction — Game rule

Form	Your commitment	Player to move	Goal
$P \wedge Q$	TRUE	Tarski's World	Choose one of $P$ , $Q$ that is false.
	FALSE	you	



## Disjunction — Truth table

P	Q	$P \vee Q$
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

## Disjunction — Game rule

Form	Your commitment	Player to move	Goal
$P \vee Q$	TRUE	you	Choose one of $P$ , $Q$ that is true.
	FALSE	Tarski's World	

# Formalisation

- Sometimes, natural language double negation means logical single negation
- The English expression **and** sometimes suggests a temporal ordering; the FOL expression  $\wedge$  never does.
- The English expressions **but**, **however**, **yet**, **nonetheless**, and **moreover** are all stylistic variants of **and**.
- Natural language disjunction can mean **inlusive-or** ( $\vee$ ) or **exclusive-or**:  $A \text{ xor } B \Leftrightarrow (A \vee B) \wedge (\neg A \vee \neg B)$

# Logical necessity

A sentence is

- **logically necessary**, or **logically valid**, if it is true in all circumstances (worlds),
- **logically possible**, or **satisfiable**, if it is true in some circumstances (worlds),
- **logically impossible**, or **unsatisfiable**, if it is true in no circumstances (worlds).

Logically possible



Logically and physically possible



Logically impossible

$$P \wedge \neg P$$

$$a \neq a$$

Logically necessary

$$P \vee \neg P$$

$$a = a$$

# Logic, Boolean logic and Tarski's world

A sentence is

- **logically necessary**, or **logically valid**, if it is true in all circumstances (worlds),
- **TW-necessary**, if it is true in all worlds of Tarski's world,
- a **tautology**, if it is true in all valuations of the atomic sentences with  $\{\text{TRUE}, \text{FALSE}\}$ .

